Free and co-free constructions for Hopf categories

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Abstract

Given any coalgebra one can allways embed it into a Hopf algebra satisfying a universal property, the so called Hopf envelope. This was shown by Takeuchi in the 70s and later extended to bialgebras by Manin in the late 80s. More generally the categories of vector spaces, coalgebras, algebras, bialgebras and Hopf algebras can be arranged in a diagram with forgetful functors between them. These all have either left adjoints (Vect-Alg, Coalg-Bialg), right adjoints (Vect-Coalg, Alg-Bialg) or both (Bialg-Hopf), where the left adjoint is the Manin construction and the right adjoint was given by Agore in 2010.

The subject of our research is it to lift this diagram to a multi-object analogue working over a category $\mathcal{V}$. We give sufficient conditions for the existence or explicitely construct the left and/or right adjoints for the forgetful functor between the categories of $\mathcal{V}$-graphs, comonoid-graphs, $\mathcal{V}$-categories, semi- and Hopf $\mathcal{V}$-categories. Particularly we construct a free and cofree Hopf-categories extending the Hopf envelope. Further we proof (co)completeness, (co-)well-poweredness as well as existence of (co-)generating familes for the involved categories extending work of Vasilakopoulou.

References

[1] A. Agore, Categorical constructions for Hopf algebras, unpublished (to appear in *Communications in Algbra*)

[2] A. Agore, Limits of coalgebras, bialgebras and Hopf algebras, *proceedings of the american mathematical society*, **139** (3), March 2011, 855-863.

[3] Manin, Yu. I. Quantum groups and noncommutative geometry. Universite de Montreal, Centre de Recherches Mathematiques, Montreal, QC, 1988.

[4] Takeuchi, M. Free Hopf algebras generated by coalgebras. *J. Math. Soc. Japan*, **23** (1971), 561–582.

[5] C. Vasilakopoulou, Enriched Duality in couble categories: $\mathcal{V}$-categories and $\mathcal{V}$-cocategories, *Journal of Pure and Applied Algebra* **223** (7), July 2019, 2889–2947

Keywords

Hopf envelopes, Hopf algebras, Hopf categories, Adjoints, multi-object structures