Hopf-Galois extensions and twisted Hopf algebroids

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Abstract

We show that the Ehresmann-Schauenburg bialgebroid of a quantum principal bundle P or Hopf Galois extension with structure quantum group H is in fact a left Hopf algebroid $L(P, H)$. We show further that if H is coquasitriangular then $L(P, H)$ has an antipode map S obeying certain minimal axioms. Trivial quantum principal bundles or cleft Hopf Galois extensions with base B are known to be cocycle cross products $B\#_\sigma H$ for a cocycle-action pair $(\triangleright, \sigma)$ and we look at these of a certain 'associative type' where $\triangleright$ is an actual action. In this case also, we show that the associated left Hopf algebroid has an antipode obeying our minimal axioms. We show that if L is any left Hopf algebroid then so is its cotwist L^ς as an extension of the previous bialgebroid Drinfeld cotwist theory. We show that in the case of associative type, $L(B\#_\sigma H, H) = L(B\#H)^{\tilde{\sigma}}$ for a Hopf algebroid cotwist $\varsigma = \tilde{\sigma}$. Thus, switching on σ of associative type appears at the Hopf algebroid level as a Drinfeld cotwist. We view the affine quantum group $U_q(\hat{\mathfrak{sl}_2})$ and the quantum Weyl group of $u_q(\mathfrak{sl}_2)$ as examples of associative type.