Hopf25

Eliezer Batista
Federal University of Santa Catarina, Brazil
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Globalization and the biactegory of partial modules

Often in the mathematical literature, constructions originally designed for groups are generalized to the context of Hopf algebras. Partial actions of groups, for example, first appeared to describe a class of operator algebras which are graded by the integers as a new kind of crossed product. In fact, partial group actions became increasingly important as a way of describing symmetries that are partially defined. Promptly, the notions of partial actions and partial representations were incorporated within the context of Hopf algebras. Far from being an exotic subject, highlighting only generalization for the sake of mere generalization, partial representations of Hopf algebras opened a skylight, shedding light on the garden of representation theory and allowing us to see new and unusual paths.

The aim of this work is to explore deeper connections between the theory of partial modules of Hopf algebras and the theory of module categories of monoidal categories. In fact, the category of partial modules over a Hopf algebra H, henceforth denoted by ${}_H\mathsf{PMod}$, is a monoidal category itself and it is a module category (also called an actegory) over the monoidal category of H-modules, ${}_H\mathsf{Mod}$. Moreover, for each partial H-module M, the left and right tensor product by M define two functors from ${}_H\mathsf{Mod}$ to ${}_H\mathsf{PMod}$ which are left adjoints. We call their associated right adjoints inner homs, denoted respectively by $\{M, -\}$ and $[M, -]$.

Given a partial H-module M, we call dilation of M a global H-module N with a linear projection T in N satisfying certain special properties such that the action of H restricted to the image $T(N)$ coincides with the original partial H-module structure on M. Every partial H-module M admits a standard globalization $\overline{M}$ which is, in certain sense, its minimal dilation and this standard globalization defines a monoidal functor D between ${}_H\mathsf{PMod}$ and ${}_H\mathsf{Mod}$. We prove that there is a natural monomorphism between the standard dilation functor D and another dilation functor obtained from an inner hom $\{A_{par}, -\}$, in which A_{par} is the monoidal unit in the category ${}_H\mathsf{PMod}$. We analyse necessary and sufficient conditions for these functors to be naturally isomorphic, this is the case, for example for finite dimensonal pointed Hopf algebras.