Hopf25

Sebastian Halbig
University of Marburg, Germany
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A non-semisimple version of the Kitaev model

In 1997, Alexei Kitaev proposed a foundational model for fault-tolerant quantum computation based on complex semisimple Hopf algebras. Its key feature is a topologically invariant code space. It is constructed using combinatorial data encoded by a graph embedded into a closed oriented surface, ensuring robustness against a wide range of errors. Beyond applications in quantum computing, the model has remarkable connections with combinatorics, the study of mapping class groups, Hopf algebra representation theory, and topological quantum field theories.

In this talk, based on joint work with A. Hirmer, U. Krähmer, C. Meuburger, and T. Voss, we present a generalisation of the Kitaev model to arbitrary finite-dimensional Hopf algebras. Two challenges prevent a straightforward approach. First, the extended Hilbert space, a Yetter–Drinfeld module whose maximal trivial submodule is the code space, relies on an involutive antipode — a condition equivalent to the underlying Hopf algebra being semisimple. Second, topological invariance is proven using projectors assembled from (co)integrals. Since we do not have these tools at our disposal, we follow a new approach, inspired by homological considerations. We introduce involutive anti-Hopf bimodules, which are related to coefficients of Hopf cyclic cohomology and allow us to form appropriate, Yetter–Drinfeld valued, variants of extend Hilbert spaces. Instead of considering trivial submodules, the analoga of the code spaces arise as bitensor products — a combination of cotensor and tensor products. Our proof of their topological invariance relies on a notion of excision and uses actions of a group related to mapping class groups. Towards computing bitensor products, we discuss induction-restriction type identities, which are particularly useful for small quantum groups. Several intriguing open questions arising from our approach will be emphasised.