Hopf25

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Parallel sessions, Forum E
Tuesday, April 22, 2025
16h30-16h55

Generalized Kac-Paljutkin algebras

The aim of this talk is to show how to construct a family of non-trivial semisimple Hopf algebras $H_{n,m}$ of dimension $n^m m!$ over a field $\mathbb{K}$ containing a primitive nth root of unity, for integers $n, m \geq 2$. The well-known eight-dimensional Kac-Paljutkin algebra arises as $H_{2,2}$, while the Hopf algebras constructed by Pansera correspond to the cases $H_{n,2}$. Each algebra $H_{n,m}$ is an extension of the symmetric group algebra $\mathbb{K}S_m$ by the m-fold tensor product $\mathbb{K}\mathbb{Z}_n^{\otimes m}$ of the group algebra of the cyclic group of order n, and can be realized as a crossed product $H_{n,m} = \mathbb{K}\mathbb{Z}_n^{\otimes m} \#_\gamma S_m$. More generally, considering the set of transpositions $s_i = (i, i+1) \in S_m$, we provide sufficient conditions on a twist J for a bialgebra B to construct a family of twists $J_{s_1}, \ldots, J_{s_{m-1}}$ for the bialgebra $R = B^{\otimes m}$. We show that the skew monoid algebra $R\#M$, where M is the free monoid on the generators $\{\overline{s}_1, \ldots, \overline{s}_{m-1}\}$, admits a bialgebra structure with comultiplication given by $\Delta(\overline{s}_i) = J_{s_i}(\overline{s}_i \otimes \overline{s}_i)$. Additional conditions on J ensure the existence of a Hopf algebra quotient $H = (R\#M)/I$, which is isomorphic to a crossed product $R\#_\gamma S_m$, and is semisimple whenever R is. We also present a family of irreducible m-dimensional representations of $\mathbb{K}\mathbb{Z}_n^{\otimes m} \#_\gamma S_m$ that are inner faithful as R-modules and exhibit a nontrivial action on a quantum polynomial algebra.