Hopf25

Shahn Majid
Queen Mary University, London, UK
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Braided Lie algebras of quantum doubles

The talk will be based on joint work with L. McCormack. We revisit the notion of braided Lie algebras introduced 30 years ago as an abstract Lie object generating quantum groups such as $U_q(\mathfrak{g})$. The notion makes sense in any braided category and consists of a coalgebra L in the category and a Lie bracket $[-,-]$ from the tensor square of L to L obeying certain axioms such as a pentagonal Jacobi identity. Every braided Lie algebra gives rise to a solution of the braided relations on L called its fundamental braiding. In the category of sets with trivial braiding, a braided Lie algebra reduces to a rack and the fundamental braiding is the rack braiding, but the concept is much more general. Among the new results, we address a long-standing issue that the braided bialgebra $U(L)$ is never a braided Hopf algebra by means of two braided Hopf algebras $U^S(L)$ and $H(L)$ related to it. We show that for the q-$\mathfrak{sl}_n$ braided Lie algebras, these land on the transmutations of $\mathbf{C}_q[\mathrm{GL}_n]$ and $\mathbf{C}_q[\mathrm{SL}_n]$ respectively. We also introduce a new construction for a class of braided Lie algebras that go significantly beyond previous ones obtained by transmutation, and use this to a describe new and simpler braided Lie algebra for $U_q(\mathfrak{sl}_n)$. Braided Lie algebras are also known to arise as the duals of the invariant 1-forms for a bicovariant calculus on a coquasitriangular Hopf algebra and we describe some examples from the Drinfeld codouble of a finite group. Finally, we introduce a natural generalisation of a braided Lie algebra and show an example where one of these arises from a differential calculus that is left-covariant but not bicovariant.