
Hopf25

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Geometric categorification of Verma modules: Grassmannian Quiver Hecke algebras

The talk is based on arXiv:2405.20262.

Classical KLR (quiver-Hecke) algebras categorify $U_q^-(\mathfrak{g})$, and their cyclotomic quotients categorify the simple modules over $U_q(\mathfrak{g})$. Naise and Vaz introduced an extension of KLR algebras that categorifies Verma modules. The goal of this work is to propose a geometric construction of these extensions, inspired by the geometric constructions of classical KLR algebras developed by Varagnolo-Vasserot and Rouquier.

To illustrate our approach, we start with the nil-Hecke algebra, which corresponds to the version \mathfrak{sl}_2 of KLR algebras. Its geometric construction relies on the variety of pairs of flags. We extend this construction by adding two Grassmannians, yielding a larger algebra than required. However, one can consider a smaller variety (with a single Grassmannian instead of two), whose homology matches the desired algebra. This smaller variety, however, lacks an evident geometric product, necessitating an intermediate construction involving two Grassmannians.

In the general case of KLR algebras, the situation is even more intricate. The final variety is constructed through multiple steps, and its definition may seem relatively complex. The method of diagrammatic varieties plays a central role in this construction. The main idea of “diagrammatic varieties” is to draw a diagram, assign vector spaces to the regions of the diagram, and define a variety based on algebraic conditions derived from the diagram. This technique provides a powerful tool for identifying the geometric conditions needed to construct the required varieties. More generally, this approach serves as a valuable framework in other contexts of geometric representation theory.