Hopf25

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Hopf braces and semi-abelian categories

The notion of semi-abelian category was introduced in [5] in order to give a categorical "generalization" for categories like groups and Lie algebras. Semi-abelian categories provide a good categorical framework to study (co)homology of non-abelian structures and develop an approach to commutator and radical theories. Moreover, in a semi-abelian category, there are natural notions of semi-direct product, internal action and crossed module.

Hopf braces were introduced in [1] as a Hopf-theoretic generalization of skew braces [4]. Under the assumption of cocommutativity, Hopf braces are equivalent to matched pairs of actions on Hopf algebras, that can be used to produce solutions of the quantum Yang–Baxter equation.

The talk is based on a recent joint work with Marino Gran [2]. First we prove that the category of cocommutative Hopf braces is semi-abelian and strongly protomodular. Moreover, under the assumption that the base field is algebraically closed and has zero characteristic, we show that the full subcategories of "primitive Hopf braces" and of "skew braces" form an hereditary torsion theory in the category of cocommutative Hopf braces, and that "skew braces" are also a Birkhoff subcategory and a localization of the latter category. Finally, we describe commutators and central extensions for cocommutative Hopf braces. The semi-abelianness of the category of cocommutative Hopf algebras achieved in [3] is recovered by considering the Birkhoff subcategory of trivial cocommutative Hopf braces.

References

[1] I. Angiono, C. Galindo, L. Vendramin, Hopf braces and Yang-Baxter operators, Proc. Amer. Math. Soc. 145 (2017), no. 5, 1981–1995.
[2] M. Gran, A. Sciandra, Hopf braces and semi-abelian categories, preprint arXiv:2411.19238.
[3] M. Gran, F. Sterck, J. Vercruysse, A semi-abelian extension of a theorem by Takeuchi, J. Pure Appl. Algebra 223 (2019), no. 10, 4171–4190.
[4] L. Guarnieri, L. Vendramin, Skew braces and the Yang–Baxter equation, Math. Comp. 86 (2017), no. 307, 2519–2534.
[5] G. Janelidze, L. Márki, W. Tholen, Semi-abelian categories, J. Pure Appl. Algebra 168 (2002), no. 2-3, 367–386.