Hopf25

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Exact module categories over $\mathrm{Rep}(u_q(\mathfrak{sl}_2))$

Andruskiewitsch and Mombelli (2007) have established a general theory of module categories over the representation category $\mathrm{Rep}(H)$ of a finite-dimensional Hopf algebra H: For an indecomposable exact module category $\mathcal{M}$ over $\mathrm{Rep}(H)$, there exists a right H-simple H-comodule algebra A with trivial coinvariants such that $\mathcal{M}$ is equivalent to $\mathrm{Rep}(A)$. The Taft algebra T_q at a root of unity q is one of the simplest examples of pointed Hopf algebras. Indecomposable exact module categories over $\mathrm{Rep}(T_q)$ have been classified by Etingof and Ostrik (2004). The small quantum group $H = u_q(\mathfrak{sl}_2)$ at a root of unity q of odd order would be the next simplest example of pointed Hopf algebras (after the Taft algebra) and has applications in various areas. For the coradically graded Hopf algebra U of H, Mombelli (2010) has already classified right U-simple U-comodule algebras with trivial coinvariants, and consequently classified exact module categories over $\mathrm{Rep}(U)$. Since $\mathrm{Rep}(U)$ and $\mathrm{Rep}(H)$ are categorically Morita equivalent, one can (in principle) obtain a list of indecomposable exact module categories over $\mathrm{Rep}(H)$ from Mombelli's list.

In this talk, we will give an explicit list of right H-simple H-comodule algebras with trivial coinvariants. The strategy is as follows. First, we note that H is a 2-cocycle deformation of U and we can explicitly write down such a 2-cocycle σ. For a (right U-simple) U-comodule algebra A, we can deform the algebra structure of A by using σ, which we denote by ${}_\sigma A$. Then one sees that the resulting algebra ${}_\sigma A$ is a (right H-simple) H-comodule algebra. Moreover, $\mathrm{Rep}({}_\sigma A)$ is the indecomposable exact module category over $\mathrm{Rep}(H)$ corresponding to the indecomposable exact module category $\mathrm{Rep}(A)$ over $\mathrm{Rep}(U)$ under the categorical Morita equivalence between $\mathrm{Rep}(H)$ and $\mathrm{Rep}(U)$. Therefore, for each A in Mombelli's list, we will give an explicit description of ${}_\sigma A$. We note that the determination of the σ-cocycle deformation ${}_\sigma A$ of A is not a trivial problem. For example, there is a 3-parameter family $A(\alpha, \beta, \lambda)$ $(\alpha, \beta, \lambda \in \mathbb{C})$ of U-comodule algebras generated by a single element w subject to $w^N = \lambda$ and such that the U-coaction is given as $w \mapsto (\alpha x + \beta y) \otimes 1 + g^{-1} \otimes w$, where N is the order of q and $U = \langle x, y, g^{\pm 1} \rangle$. After the 2-cocycle deformation, the algebra ${}_\sigma A(\alpha, \beta, \lambda)$ is still generated by a single element w and a similar H-coaction. However, the minimal polynomial of w becomes more complicated:

$$\prod_{i=0}^{N-1} w - (\xi_+ q^{2i} + \xi_- q^{-2i}),$$

where $\xi_\pm \in \mathbb{C}$ are chosen so that they satisfy $\xi_+^N + \xi_-^N = \lambda$ and $\xi_+ \xi_- (1 - q^2) = \alpha\beta$.

This talk is based on ongoing joint work with Kenichi Shimizu (Shibaura Institute of Technology) and Daisuke Nakamura (Okayama University of Science).