Hopf25

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Simple algebras and exact module categories

Exactness is an important property of module categories over tensor categories, and module categories are often understood via categories of modules for an algebra object. Thus, it is natural to ask when a category of modules is an exact module category. Etingof and Ostrik conjectured that, in the indecomposable case, exactness of the category of modules is equivalent to the absence of ideal objects in the algebra object.

Indeed, exactness of the internal Hom renders every module "internally projective-injective", so the algebra should be "internally semisimple", and indecomposability implies that it must be simple. In this talk, I will give a proof of the conjecture, based on an analogue of the Jacobson radical inside an algebra object.

I will explain how this result generalizes the results of Skryabin on projectivity of Hopf modules for simple comodule algebras over a finite-dimensional Hopf algebra, and show some applications to tensor categories in positive characteristic, and to vertex operator algebras.

Time allowing, I will discuss the notions of "internally semisimple" algebras and modules which emerge from the proof, and a characterization of modules over the maximal semisimple quotient of an algebra.