QUADRATIC ALGEBRAS OF LEFT NON-DEGENERATE SET-THEORETIC SOLUTIONS OF THE YANG-BAXTER EQUATION

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In 1992 Drinfel'd proposed to investigate so called set-theoretic solutions of the Yang-Baxter equation. Recall that these are tuples (X, r) with X a nonempty set and on X^3

$$(r \times id_X)(id_X \times r)(r \times id_X) = .(id_X \times r)(r \times id_X)(id_X \times r).$$

The structure monoid and group encoding these solutions, the respective free algebraic objects generated by X quotiented by the defining relations $xy = uv$ if $r(x, y) = (u, v)$, were introduced by Gateva-Ivanova and Van den Bergh, and Etingof, Schedler and Soloviev. They further investigated their (semi)groupalgebras and showed that for finite involutive non-degenerate solutions these algebras are noetherian of finite Gelfand-Kirillov dimension bounded by the size of X that satisfy a polynomial identity. Moreover, it was shown that these algebras enjoy nice homological properties similar to polynomial rings in several commuting variables. We report on the state of the art and recent findings in several joint works with I. Colazzo, E. Jespers, L. Kubat and C. Verwimp. We discuss when these algebras are (left) Noetherian, their Gelfand-Kirillov dimension and a characterization of said homological properties in terms of the solution.In particular, we mention the importance of the divisibility structure of the structure monoid in several proofs and the characterization of prime ideals.