THE POISSON–FOURIER TRANSFORM FOR BICROSSED PRODUCTS

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The quantum duality Principle of Drinfel'd states that any quantization $\mathcal{G}_\hbar$ of a Poisson–Lie group $\mathcal{G}$ should be dual as a quantum group to a quantization $\mathcal{G}^*_\hbar$ of the Poisson dual group $\mathcal{G}^*$. In this talk we will make sense of this principle for semi-direct products ($\mathcal{G} = G \ltimes V, \mathcal{G}^* = H \ltimes W$) with V, W abelian, where we can realise the quantizations $\mathcal{G}_\hbar$ and $\mathcal{G}^*_\hbar$ as locally compact quantum groups through a bicrossed product between G and H. A key role is played by suitable measure class isomorphisms $\eta_G : G \to \hat{W}$ and $\eta_H : H \to \hat{V}$ which we call *abelian approximations*. These give rise to a unitary operator $\mathcal{F}_\mathcal{G} : \mathrm{L}^2(\mathcal{G}) \to \mathrm{L}^2(\mathcal{G}^*)$, which induces an isomorphism of locally compact quantum group $\mathcal{F}_\mathcal{G} : \hat{\mathcal{G}}_\hbar \cong \mathcal{G}^*_\hbar$. We say that $\mathcal{F}_\mathcal{G}$ is the *Poisson–Fourier transform* between $\mathcal{G}$ and $\mathcal{G}^*$, and we understand the above isomorphism as implementing the quantum duality principle. After recalling elements of the theory of locally compact quantum groups and the bicrossed product construction in this setting, we discuss abelian approximations and the resulting Poisson–Fourier transform. We conclude with a strategy to produce abelian approximations from suitable Lie bialgebras, together with the corresponding unitary dual 2-cocycle or quantum R-matrix in the triangular and quasi-triangular case.