$\mathsf{SL}(k)$-FRIEZES FROM CLUSTER ALGEBRAS

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Abstract:
Classical frieze patterns are combinatorial structures which relate back to Gauss' Pentagramma Mirificum, and have been extensively studied by Conway and Coxeter in the 1970's.
A classical frieze pattern, or $\mathsf{SL}(2)$-frieze, is an array of numbers satisfying a local (2×2)-determinant rule. Conway and Coxeter gave a beautiful and natural classification of $\mathsf{SL}(2)$-friezes via triangulations of polygons. One way to generalise this idea is to ask for such an array to satisfy a $(k \times k)$-determinant rule , for k at least 2, leading to the notion of $\mathsf{SL}(k)$-friezes. When k is at least 3 these friezes are not yet well understood.
In this talk, we'll discuss how one might start to fathom $\mathsf{SL}(k)$-friezes. The links between friezes and cluster combinatorics which is encoded by triangulations of polygons in the $k = 2$ case suggests a link to Grassmannian varieties under the Plücker embedding, and the cluster algebra structure on their homogeneous coordinate rings. We find a way to exploit this relation for higher $\mathsf{SL}(k)$-friezes and provide an easy way to generate $\mathsf{SL}(k)$-friezes via Grassmannian combinatorics. This talk is based on joint work with Baur, Faber, Serhiyenko and Todorov.